Correction to:

Global algebraic K-theory

Journal of Topology 15 (2022), 1325-1454

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Donald Yau has pointed out an issue with Construction 3.17 concerning the definition of the morphism of symmetric spectra ψ_Y^G, and some potentially misleading typos; the problem with Construction 3.17 is that no value of this morphism at the empty set is specified. The origin of the problem is that the values of the symmetric spectrum $Y\langle\mathbb{S}\rangle$, for Y a Γ-$\mathcal{M}$-category, were defined in Construction 3.3 by a case distinction, with the value at the empty set defined as the realization of the full subcategory of $\mathcal{M}$-fixed objects in $Y(1_+)$.

Unravelling all definitions yields that the values of the symmetric spectra $(F^G Y)\langle\mathbb{S}\rangle$ and $F^G(Y\langle\mathbb{S}\rangle)$ – the source and target of the morphism ψ_Y^G – are, respectively:

- $(F^G Y)\langle\mathbb{S}\rangle(\emptyset)$ is the realization of the full subcategory of $\mathcal{M}$-fixed objects in $(Y(1_+)[\omega^G])^G$;
- $F^G(Y\langle\mathbb{S}\rangle)(\emptyset)$ is the realization of the full subcategory of $\mathcal{M}$-fixed objects in $Y(1_+)$.

In the generality of Construction 3.17 and Theorem 3.20, the former need not be contained in the latter, and I see no reasonable map from the former to the latter.

The fix to this problem is to **add the hypothesis** to Construction 3.17 and Theorem 3.20 that the underlying $\mathcal{M}$-category $Y(1_+)$ of the Γ-$\mathcal{M}$-category Y **is tame**. Assuming this, the lemma below applied to $\mathcal{C} = Y(1_+)$ shows that $(F^G Y)\langle\mathbb{S}\rangle(\emptyset)$ and $F^G(Y\langle\mathbb{S}\rangle)(\emptyset)$ are equal, and we can define the morphism ψ_Y^G at the empty set to be the identity of this space.

Theorem 3.20 is invoked twice in the paper, and in each case for parsummable categories, which are tame by definition. So the remaining results of the paper are still valid with the additional tameness hypothesis on $Y(1_+)$ in Construction 3.17 and Theorem 3.20.

We let G be a finite group. Then as in Construction 2.21, we let $G \times \mathcal{M}$ act on the set ω^G of functions from G to $\omega = \{0, 1, 2, \dots\}$ by pre- and postcomposition, i.e., by

$$((g,u)\cdot f)(h) \ = \ u(f(g^{-1}h)) \ ,$$

where $(g,u) \in G \times \mathcal{M}$, $f : G \to \omega$ and $h \in G$.

Lemma. Let $\mathcal{C}$ be a tame $\mathcal{M}$-category and G a finite group. Then every object of $\mathcal{C}[\omega^G]$ that is fixed by the action of $G \times \mathcal{M}$ is already fixed by all self-injections of ω^G. Hence the $\mathcal{M}$-fixed objects of $F^G\mathcal{C} = \mathcal{C}[\omega^G]^G$ coincide with the objects of $\mathcal{C}[\omega^G]$ that are fixed by all self-injections of ω^G.

Proof. Let x be an object of $\mathcal{C}[\omega^G]$ that is fixed by $G \times \mathcal{M}$. The tameness hypothesis ensures that x is supported on some finite subset A of ω^G. We choose an injection u in $\mathcal{M}$ such that $u^G(A)$ is disjoint from A. Since $x = (u^G)_*(x)$, the object x is supported both on A and on $u^G(A)$. The proof of Proposition 2.13 (i) shows that then x is also supported on their intersection of A and on $u^G(A)$. Since these two sets are disjoint, x is supported on the empty set, i.e., it is fixed by all self-injections of ω^G. $\square$

Typos:

• In the first displayline of Definition 1.1., the codomain of the structure map i_* needs to be $X(B)$ (and not S^B)

• In the proof of Corollary 7.7, the second sentence should start with 'Because $\lambda_\sharp \circ \kappa = \lambda_\flat$' (as opposed to $\lambda_\flat \circ \kappa = \lambda_\sharp$)

• In Construction 11.1, in the definition on the isomorphism $u^a_\circ$ on page 1440, the sentence 'Then we choose a permutation σ (...)' is missing the condition that $\sigma(i) = u(i)$ only for those $i \in \{0, \ldots, n\}$ such that $a_i \neq 0$.