Minisymposium 25 - Inverse Probleme und Inkorrektheits-Phänomene

Approximate Solutions to Inverse Problems for Elliptic Equations

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In this contribution we study Cauchy problems for 2-d. elliptic partial differential equations. These consist in determining a function – and its normal derivative – on one side of a rectangular domain from Cauchy data on the opposite side. With Cauchy data we mean the function itself and its normal derivative. On the other parts of the boundary Dirichlet or Neumann data are given. This type of problems is known to be illposed. Hadamard gave a classical example in 1923 demonstrating its illposedness. It should be noted that such Cauchy problems are conditionally well-posed which means that under certain restrictions on the data the problems are well-posed. Approximate solutions can be obtained by a semidiscretization of the rectangular domain which leads to a coupled system of boundary value problems for ordinary differential equations. The system can be decoupled by solving an eigenvalue problem in advance.
We study its stability (with respect to perturbations in the data) as well as the error behaviour in terms of the discretization parameter (for the semidiscretization).

References:

Charton, M., Reinhardt, H.-J.: Approximation of Cauchy problems for elliptic equations using the method of lines, WSEAS Transactions on Math., 4/1 (2005), 64-69.