Minisymposium 21 - Automorphic forms and their applications

The theta divisor and its “square”

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Let C be a smooth projective curve over an algebraically closed field k and let (X, Θ) be its Jacobian variety X, with the associated theta divisor Θ. Let δ_Θ denote the intersection cohomology sheaf of Θ, a perverse sheaf on X. Then the convolution product

$$\delta_\Theta * \delta_\Theta$$

is a sheaf complex on X. It is a direct sum $\bigoplus_{A,\mu} m(\mu, A) \cdot A[\mu]$ of translates of irreducible perverse sheaves A on X with certain multiplicities $m(\mu, A)$. By definition the coefficients of $\delta_\Theta * \delta_\Theta$ are those A, for which the multiplicity $m(\mu, A)$ is nonzero for some $\mu \in \mathbf{Z}$. The coefficients A are sheaf complexes on X. Let $\mathcal{H}^\nu(A)$ denote their associated cohomology sheaves on X for $\nu \in \mathbf{Z}$. Let $\kappa \in X(k)$ be the Riemann constant defined by $\Theta = \kappa - \Theta$ (it depends on the choice of the Abel-Jacobi map $C \to X$). Then we show

Theorem *For a curve C of genus $g \geq 3$ there exists a unique irreducible perverse sheaf A among the coefficients of $\delta_\Theta * \delta_\Theta$, characterized by one of the following equivalent properties*

(1) $\mathcal{H}^{-1}(A)$ *is nonzero, but not a constant sheaf on* X.
(2) $\mathcal{H}^{-1}(A)$ *is a skyscraper sheaf on* X.
(3) $\mathcal{H}^{-1}(A)$ *is the skyscraper sheaf* $H^1(C) \otimes \delta_{\{\kappa\}}$ *concentrated in* $\kappa \in X$.

and the support of this perverse sheaf A is a translate of $\kappa + C - C$ in X.

Torelli’s theorem is an immediate consequence. We also show, that higher convolution products of δ_Θ and δ_C essentially are perverse sheaves on X (i.e. they are perverse up to translates of constant sheaves on X).